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STOCHASTIC SYSTEMS

A Stochastic Estimation Problem at Aeromagnetometer Deviation Compensation

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Abstract—A formulation of the problem of magnitometer deviation compensation occurring at aeromagnetic survey is considered in the form of a standard stochastic estimation problem. A specific feature of this approach is the introduction of the model of a geomagnetic field anomaly. The parameters of the stochastic model are selected basing on spectral and variance analysis of the aeromagnetic survey data. Normalization of the problem parameters is made; this makes it possible to estimate the accuracy of compensation and conduct a necessary decomposition. The posed stochastic estimation problem is solved by the method of Kalman filtering.

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1. INTRODUCTION

A magnetic method of geophysical exploration solves the following geological problems: study of platform foundation, identification and tracking of faulting tectonic dislocations, searching for ore deposits, and etc. The method is based on measurement of geomagnetic field modulus, numerical treatment of measurements, and geological interpretation of the obtained results.

The aim of aeromagnetic survey activities is to define an anomalous component of the geomagnetic field. The anomalous component is a deviation of the value of the geomagnetic field from the normal one which is a field of the homogeneously magnetized globe and additional dipoles in the core justifying continental anomalies. The anomalous part of the constant geomagnetic field has the information about the geological structure of the upper strata of the Earth crust.

Today accuracy requirements to aeromegnetic survey data are about 10^{-5} to the quantity of the magnetic field $\sim 50\,000$ nT (nanotesla). The constraint is connected to a great extent to technical feasibilities of the existing survey equipment; however, there are already sensors with a considerably greater accuracy of measurements [1]; hence, the design of an algorithm of a more complex and qualitative treatment becomes an important problem.

In conducting a magnetometer survey with the use of mobile carriers (airplane or helicopter), the sensor of the equipment is affected not only by the geomagnetic field but also by the field of the carrier itself which causes corruption (deviation) of measurement results. The problem of eliminating the influence of the field of the carrier can be solved by the method of deviation compensation.

The approaches to solving the problem are properly studied and are based on the representation of magnetic deviation by a sum of the fields justified by constant, inductive, and eddy sources for the description of which is used the Poisson model [2, 3]. Sometimes the information about the altitude of the carrier flight is used for taking the vertical gradient of the geomagnetic field into account.

There are some commercial programs that realize similar algorithms of compensation. The most known programs are that of the Canadian companies “Pico Envirotec” and “RMS Instruments” [4]. There is a Russian analog developed in the Federal State Unitary Scientific Production Enterprise “Geologorazvedka” [3].

It is to be noted that the approach proposed in the paper differs from the ones known before first of all by the introduction of a stochastic model of the anomalous field. Today the majority of producers do not select all parameters of the magnetic field in data processing. As a consequence, some algorithms work out their own parameters of deviation for each direction of the flight. Developers of “Geologorazvedka” estimate the value of the vertical gradient of the field in their algorithms [3]. By introducing the model of the anomalous field, we can bear in mind not only the influence of the vertical but also of the horizontal gradient of the field. The character of this influence was described in the 1960s [5]. The parameters of deviation obtained as a result of solving this problem are applicable for any directions of the flight.

The idea of introducing the stochastic model of the anomalous field is successfully used by the laboratory of control and navigation at Moscow State University in solving the problem of aerogravimetry [6]. Since the modulus of the magnetic field, like the potential of the gravitational field, satisfies the Laplace equation in the air, a similar approach was proposed in solving the problem of aeromagnetometer deviation compensation.

The aim of the paper is to formulate and solve the linear stochastic estimation problem in defining parameters of aeromagnetometer deviation.

2. FORMULATION OF THE PROBLEM

The procedure of compensating the carrier influence requires a calibration flight that is necessary for defining parameters of deviation. The calibration flight assumes the aircraft climb up to the maximal altitude of $h \sim 1000$ m to minimize the influence of the anomalous geomagnetic field. A series of evolutions with angles of the order $5°$ in hunting, roll, and pitch on four different courses necessary for measuring the vector of magnetic field at different orientations with respect to the carrier. After defining the parameters of noise, we can conduct a magnetometer survey at the operating altitude of $h \sim 100$ m.

To solve the compensation problem, two magnetometers are used. The first magnetometer is quantum; it allows measuring the modulus B of the magnetic field with high accuracy. The second one is a ferroprobe with less accuracy; it is necessary for obtaining the vector $\overline{B}_F$ of the magnetic field.

In solving the compensation problem, together with the above measurements of the modulus and vector of magnetic field is necessary the information about the coordinates and velocity of the carrier obtained by the results of the operation of the GPS receiver. The result of solving the compensation problem are parameters of magnetic disturbance sufficient for ensuring the finite accuracy over the range of ~ 0.1 nT.

In the paper, we study the reduction of the compensation problem above to a standard formulation of the stochastic estimation problem for deviation parameters. It ie necessary to set a closed estimation problem, i.e., obtain generating equations for the anomalous field, equations for deviation parameters, and equations of measurements.

A mathematical expectation for the deviation generated by magnetic masses (obtained by Poisson in 1824) denoted as $\Delta\overline{B}_m$ has the form [2, 3]:

$$\Delta\overline{B}_m = \overline{K} + L\overline{B}_0. \tag{1}$$

Here $\overline{B}_0$ is a vector of the external field; $\overline{K}$ is a constant or hard component of the deviation specified by the field of hard magnetic materials whose magnetic moment is constant; L is a tensor of inductive or soft component conditioned by the field of soft magnetic materials.

The deviation $\Delta\overline{B}_i$ generated by inductive currents occurs at variations in time of the magnetic field, at nonuniform motions of the carrier, or at its motion in the field with great vertical and horizontal gradients [2, 3]:

$$\Delta\overline{B}_i = M\frac{d\overline{B}_0}{dt}, \tag{2}$$

where M is a matrix of current influence.

The model of measurements in the estimation problem is obtained at joint use of the model of complete deviation $\Delta\overline{B}_{sum} = \Delta\overline{B}_m + \Delta\overline{B}_i$ for the quantum sensor and ferroprobe.

To obtain a closed estimation model, the model of measurements is supplemented by differential Eqs. (7) and (8) describing components of the anomalous magnetic field and deviation parameters. In our formulation of the problem, deviation parameters are assumed constant. The field is described by a stochastic model whose parameters are selected basing on spectral and variance analysis of the real data.

3. NORMALIZATION OF PARAMETERS OF THE PROBLEM

The stored experience makes it possible to specify typical values of deviation parameters for widespread aircraft such as An-2 in Russia and Cessna Caravan abroad and conduct normalization of the model of magnetic disturbance according to [7] and its simplification taking the occurring dimensionless small parameter $\varepsilon \sim 0.1$ into account.

We shall represent typical values of main deviation parameters for the maximal and operating altitude. Typical values of variables are defined by maximal in modulus values of their quantities on the time interval under study [7].

$B_* = 50\,000$ nT is a typical value of the geomagnetic field;

$\Delta B_{m*} = 50$ nT are typical values of disturbance generated by magnetic masses;

$\Delta B_{i*} = 1$ nT are typical values of disturbance generated by the field of eddy currents;

$T_* = 5$ s is a typical time of evolution;

$V_* = 50$ m/s is a typical velocity of the aircraft;

$\nabla B_* = 1$ nT/m is a typical value of gradients of the magnetic field at $h \sim 100$ m, in strongly gradient fields it may achieve 10 nT/m;

$\nabla B_* = 0.01$ nT/m is a typical values of gradients of the magnetic field at $h \sim 1000$ m;

$T_a = B_*(\nabla B_* V_*)^{-1} = 1000$ s is a typical value of anomaly at $h \sim 100$ m, there are possible zones where $T_a = 100$ s;

$T_a = B_*(\nabla B_* V_*)^{-1} = 100\,000$ s is a typical time of anomaly at $h \sim 1000$ m.

The magnetic field, taking the deviation into account, is expressed according to the formula [3]:

$$\overline{B} = \overline{B}_0 + \Delta\overline{B}_m + \Delta\overline{B}_i = \overline{B}_0 + \overline{K} + L\overline{B}_0 + M\frac{d\overline{B}_0}{dt}.$$

Let us write out the formula for the derivative of the vector of magnetic field in time:

$$\frac{d\overline{B}_0}{dt} = \frac{d(B_0\overline{e}_0)}{dt} = \frac{dB_0}{dt}\overline{e}_0 + \overline{B}_0\frac{d\overline{e}_0}{dt},$$

where $\overline{e}_0$ is a unit vector that coincides in direction with $\overline{B}_0$. We shall introduce a notation for new normalized variables:

$$\tau = \frac{t}{T_*}, \quad \overline{b} = \frac{\overline{B}}{B_*}, \quad \overline{b}_0 = \frac{\overline{B}_0}{B_*}, \quad b = \frac{B}{B_*}, \quad b_0 = \frac{B_0}{B_*},$$

whence

$$\frac{d\overline{e}_0}{dt} = \frac{1}{T_*}\frac{d\overline{e}_0}{d\tau}, \quad \frac{dB_0}{dt} = \nabla B_* V_* \frac{db_0}{d\tau} = \frac{B_*}{T_a}\frac{db_0}{d\tau}.$$

Taking the last formulae into consideration, we normalize the expression for magnetic disturbance

$$B_*\overline{b} = B_*\overline{b}_0 + \overline{K} + LB_*\overline{b}_0 + Mb_0\frac{B_*}{T_*}\frac{d\overline{e}_0}{d\tau} + M\frac{B_*}{T_a}\frac{db_0}{d\tau}\overline{e}_0,$$

or, if to rewrite in a corresponding order

$$\overline{b} = \overline{b}_0 + \overline{k} + [l]\overline{b}_0 + [m]b_0\frac{d\overline{e}_0}{d\tau} + [m]\frac{T_*}{T_a}\frac{db_0}{d\tau}\overline{e}_0.$$

We shall estimate the order of the dimensionless vector $\overline{k}$ and components of the matrices $[l]$ and $[m]$:

$$|\overline{k}| = \frac{|\overline{K}|}{B_*} \sim \frac{\Delta B_{m*}}{B_*} \sim 10^{-3} = \varepsilon^3, \quad l_{ij} = L_{ij} \sim \frac{\Delta B_{m*}}{B_*} \sim 10^{-3} = \varepsilon^3.$$

Bearing in mind that $T_*/T_a \ll 1$ and assuming the homogeneity of conducting materials the presence of which leads to the occurrence of an eddy component of deviation, we obtain

$$\begin{aligned} \Delta B_{i*} \sim \frac{M_{ij}B_*}{T_*} &\Rightarrow M_{ij} \sim \frac{\Delta B_{i*}T_*}{B_*}, \\ m_{ij} = \frac{M_{ij}}{T_*} &= \frac{\Delta B_{i*}}{B_*} = 10^{-4} = \varepsilon^4, \\ m_{ij}\frac{T_*}{T_a} &\sim 0.5 \times 10^{-2} \times \varepsilon^4 \sim \varepsilon^6 \quad \text{at 100 m}, \\ m_{ij}\frac{T_*}{T_a} &\sim 0.5 \times 10^{-4} \times \varepsilon^4 \sim \varepsilon^8 \quad \text{at 1000 m}. \end{aligned}$$

Thereby we can rewrite the expression for magnetic disturbance taking the order of smallness of entering quantities for maximal and operating altitudes into account:

$$\overline{b} = \overline{b}_0 + \varepsilon^3\breve{\overline{K}} + \varepsilon^3\left(\breve{L} + \varepsilon^5\frac{\breve{M}}{b_0}\frac{db_0}{d\tau}\right)\overline{b}_0 + \varepsilon^4\breve{M}b_0\frac{d\overline{e}_0}{d\tau} \quad \text{at} \quad h \sim 1000 \text{ m}, \tag{3}$$

$$\overline{b} = \overline{b}_0 + \varepsilon^3\breve{\overline{K}} + \varepsilon^3\left(\breve{L} + \varepsilon^3\frac{\breve{M}}{b_0}\frac{db_0}{d\tau}\right)\overline{b}_0 + \varepsilon^4\breve{M}b_0\frac{d\overline{e}_0}{d\tau} \quad \text{at} \quad h \sim 100 \text{ m}, \tag{4}$$

where $|\breve{\overline{K}}|, \breve{L}_{ij}, \breve{M}_{ij}$ are quantities of order 1.

4. SCALAR MODEL OF MEASUREMENTS

In this section, we shall write out a scalar model of measurements of the scalar magnetometer in the expansion in small parameter (for the maximal altitude, it is of the order 1000 m) with accuracy up to ε^6 inclusive. We shall write relation (3) for the points of position of the scalar and vector (those with the index $_F$ refer to the ferroprobe) sensors jointly for the unit vectors:

$$\overline{e} = \left(\overline{e}_0 + \varepsilon^3 \breve{L}\overline{e}_0 + \varepsilon^4 \breve{M}\frac{d\overline{e}_0}{d\tau}\right)\frac{b_0}{b} + \varepsilon^3 \frac{\breve{\overline{K}}}{b},$$
$$\overline{e}_F = \left(\overline{e}_0 + \varepsilon^3 \breve{L}_F\overline{e}_0 + \varepsilon^4 \breve{M}_F\frac{d\overline{e}_0}{d\tau}\right)\frac{b_0}{b} + \varepsilon^3 \frac{\breve{\overline{K}}_F}{b}.$$

Considering these relations jointly, we can express $b = \overline{b}^T\overline{e}$ through the vectors $\overline{b}_F$ and $\overline{e}_F$.

$$b = b_0 + \varepsilon^3\overline{e}_F^T\overline{K} + 0.5\varepsilon^3\overline{e}_F^T L_S \overline{b}_F + \varepsilon^4\overline{b}_F^T M\frac{d\overline{e}_F}{d\tau} + \varepsilon^6 C + o(\varepsilon^6). \tag{5}$$

Here $L_S = L + L^T$ is a symmetric matrix, C is a scalar constant, $K = (K_i)$, $L = (L_{ij})$, $M = (M_{ij})$. We can not to consider the constant C since the aim of aerial survey activities is to define the anomalous component of the geomagnetic field. For the other components, it is possible to show in analyzing the observability of the obtained problem that it contains 16 observed parameters:

$$K_1, K_2, K_3, \Delta L_{11}, L_{12}, \Delta L_{22}, L_{13}, L_{23},$$
$$\Delta M_{11}, \Delta M_{22}, M_{12}, M_{21}, M_{13}, M_{31}, M_{23}, M_{32},$$
$$\Delta L_{ii} = L_{ii} - L_{33}, \quad \Delta M_{ii} = M_{ii} - M_{33}.$$

The remained parameters of the matrices L_S, M do not enter due to the symmetry of the matrix L_S and identities

$$\overline{e}_F^T\frac{d\overline{e}_F}{d\tau} \equiv 0, \quad \overline{e}_F^T\overline{e}_F \equiv 1.$$

It is to be noted that for the operating altitude (about 100 m), one more term of the order ε^6, as we can see from formula (4), is added in the scalar model of deviation. It is related to the time derivative of the modulus of the external field dB_0/dt conditioned by a comparatively great gradient. It is impossible to take its influence into consideration due to its smallness at calibration flight but it becomes considerable at operational altitudes. Thereby in separate cases, to increase the accuracy of compensation, it is necessary to estimate an additional term in the model. For this, it is necessary to process intersection points of survey routes that are present, as a rule.

5. STOCHASTIC MODEL OF THE FIELD

In selecting the model of the magnetic field anomaly, we shall use the ideas realized in solving problem of aerogravimetry [6]. We shall consider a model of the geomagnetic field linear in deviations from the horizontal plane for the altitudes of ~ 1000 m:

$$B_0(x, y, z) = B^h(x, y) + B_z(x, y, z_0)\Delta z, \quad B_z(x, y, z_0) = \left.\frac{\partial B_0}{\partial z}\right|_{z=z_0}. \tag{6}$$

Here B_z is a vertical gradient of the magnetic field; $\Delta z = (z - z_0)$ is a deviation from the horizontal plane $z = z_0$. We shall separately model the magnetic field B^h in the plane and vertical gradient B_z as random processes. In place of the latter, we shall choose integrals of the white noise of the 3rd and 2nd orders respectively for the field and vertical gradient. The choice of orders of the model is specified by spectral characteristics of the magnetic field which will be stipulated below.

We shall make another assumption. Let us consider horizontal projections of lines of the path rectilinear. This is admissible since lateral deviations of the aircraft are small, of the order of 5–10 m, whereas changes of the altitude amount to 100 m and more. This is correct since at altitudes of $\sim$ 1000 m, the quantity of the gradient is $\sim$ 0.01 nT/m. It follows from the rectilinearity of routes that the motion in the plane can be specified by one parameter x and the velocity of motion along the route is $V = dx/dt$. We obtain the following system of equations:

$$\frac{dB^h}{dt} = \frac{\partial B^h}{\partial x}\frac{dx}{dt} = VB^h_x,$$

$$\frac{dB^h_x}{dt} = \frac{\partial B^h_x}{\partial x}\frac{dx}{dt} = VB^h_{xx}, \quad \frac{dB^h_{xx}}{dt} = \frac{\partial B^h_{xx}}{\partial x}\frac{dx}{dt} = Vq_1,$$

$$\frac{dB_z}{dt} = \frac{\partial B_z}{\partial x}\frac{dx}{dt} = VB_{zx}, \quad \frac{dB_{zx}}{dt} = \frac{\partial B_{zx}}{\partial x}\frac{dx}{dt} = Vq_2.$$

Here q_1, q_2 are normal random process of the type of white noise. We shall rewrite equations in matrix form,

$$\dot{\overline{X}}_1 = A_1\overline{X}_1 + v\overline{q}_1, \tag{7}$$

where

$$\overline{X}_1 = \begin{pmatrix} B^h \\ B^h_x \\ B^h_{xx} \end{pmatrix}, \ A_1 = \begin{pmatrix} 0 & V & 0 \\ 0 & 0 & V \\ 0 & 0 & 0 \end{pmatrix}, \ \overline{q}_1 = \begin{pmatrix} 0 \\ 0 \\ q_1 \end{pmatrix},$$

$$\dot{\overline{X}}_2 = A_2\overline{X}_2 + v\overline{q}_2, \tag{8}$$

where

$$\overline{X}_2 = \begin{pmatrix} B_z \\ B_{zx} \end{pmatrix}, \ A_2 = \begin{pmatrix} 0 & V \\ 0 & 0 \end{pmatrix}, \ \overline{q}_2 = \begin{pmatrix} 0 \\ q_2 \end{pmatrix},$$

$$M[\overline{q}_i] = 0, \quad M[\overline{q}_i(t)\ \overline{q}_i^{\mathrm{T}}(s)] = Q_i\delta(t-s) \quad \text{at} \quad i = 1, 2,$$

$$Q_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_1^2 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_2^2 \end{pmatrix},$$

where $\delta(x)$ is a delta function, σ_1, σ_2 are root-mean-square deviations.

To estimate parameters σ_1, σ_2, we shall use dispersion relations:

$$\dot{P}_x = AP_x + P_xA^{\mathrm{T}} + Q,$$

where $P_x(t) = M[\overline{X}_i(t)\ \overline{X}_i^{\mathrm{T}}(s)]$ is a covariance matrix.

We shall estimate the parameter σ_1 for the field in the plane.

$$\frac{d}{dt}\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{pmatrix} = \begin{pmatrix} 2Vp_{12} & V(p_{22}+p_{13}) & Vp_{23} \\ V(p_{22}+p_{13}) & 2Vp_{23} & Vp_{33} \\ Vp_{23} & Vp_{33} & V\sigma_1^2 \end{pmatrix}.$$

Solving the dispersion relation at zero initial conditions, we obtain expressions for components of the matrix P_x. In particular, for p_{11}

$$p_{11} = \frac{V^5 t^5 \sigma_1^2}{20}.$$

To define the parameter σ_1, we shall specify a boundary condition on p_{11}. Proceeding from the real data, at distances of the order of 1000 m, i.e., the typical velocity of $V = 50$ m/s during the typical time of $t \sim 20$ s, the value of the field changes by the quantity of the order of 30 nT ($p_{11} \sim 30^2 \sim 10^3$ nT),

$$p_{11} = \frac{50^5 20^5 \sigma_1^2}{20} \sim 10^3 \Rightarrow \sigma_1 \sim 10^{-6}.$$

We shall make sure that the obtained model for the field in the plane with the parameter $\sigma_1 \sim 10^{-6}$ does not manifest itself at evolutions $t \sim 1$ s (on the lengths ~ 50 m), i.e., changes by the quantity of the order of noises 0.01 nT:

$$p_{11} = \frac{50^5 1^5 10^{-12}}{20} \sim 1.5 \times 10^{-5} \Rightarrow \sqrt{p_{11}} \sim 0.004.$$

Moreover, it is necessary to meet the conditions requiring the adequacy of the quantity of the horizontal gradient. Proceeding from the real data on times of evolution $t \sim 20$ s, the value of the vertical gradient changes by the quantity of the order 0.01 nT/m:

$$p_{22} = \frac{V^3 t^3 \sigma_1^2}{3} = \frac{50^3 20^3 10^{-12}}{3} \sim 3.33 \times 10^{-4} \Rightarrow \sqrt{p_{22}} \sim 0.02.$$

It is to be noted that in choosing the model of the magnetic field anomaly of a smaller order, the value of $\sqrt{p_{11}}$ considerably increases on times of evolutions $t \sim 1$ s, in particular, for the model of the second order $\sqrt{p_{11}} \sim 0.2$, which is by an order of magnitude much than that of the expected measurement noises. Thus it is shown that the choice of the model of the third order is conditioned at specified characteristic values of the anomalous magnetic field.

To analyze the model of vertical gradient, it is convenient to studying in the frequency domain, we shall apply the two-dimensional Fourier transformation:

$$\widetilde{B}_0(k_x, k_y, z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-i(k_x x + k_y y)} B_0(x, y, z) dx dy.$$

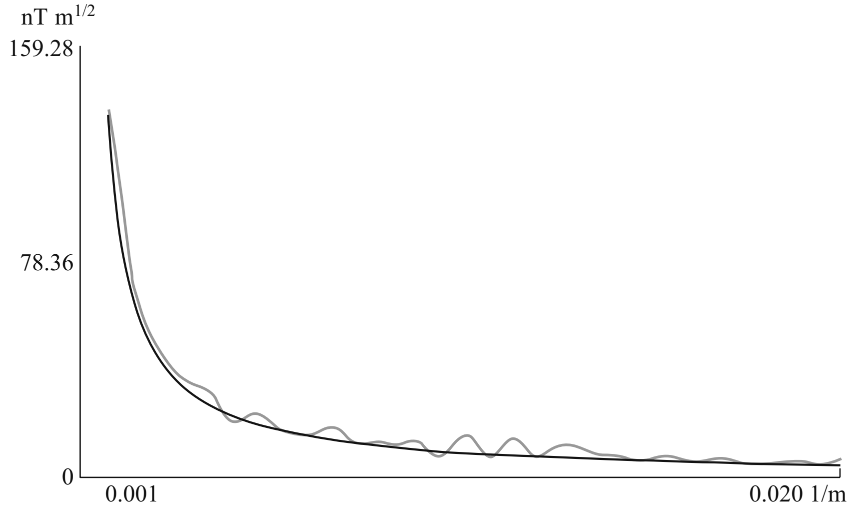
The field $B_0(x, y, z)$ in the air satisfies the Laplace equation $\partial^2 B_0/\partial x^2 + \partial^2 B_0/\partial y^2 + \partial^2 B_0/\partial z^2 = 0$. Therefore, all formulae of the reduction of anomaly along the vertical are true, in particular, for the spectrum of vertical gradient is true the equality [6]

$$\widetilde{B}_z = -\omega \widetilde{B}_0, \quad \omega = \sqrt{k_x^2 + k_y^2}.$$

Let us write the expression of the spectrum of vertical gradient taking properties of the horizontal gradient into account:

$$\widetilde{B}_x = -i k_x \widetilde{B}_0, \quad \widetilde{B}_y = -i k_y \widetilde{B}_0 \Rightarrow \widetilde{B}_z = i\sqrt{\widetilde{B}_x^2 + \widetilde{B}_y^2}.$$

It is obvious that the model of the vertical gradient in the sense of spectral density must be the same as the model of the horizontal one. Thereby specifying $\sigma_2 = \sigma_1$ and taking the model of the second order, we obtain required typical parameters of the gradient the same as, in particular, p_{22}.



Spectral density of the field in the horizontal plane at the altitude of the order of 1 km.

We shall construct diagrams of the spectral density for the field B^h (the figure) calculated for real measurements of the magnetic field (the grey color) and obtained basing on the generating equations (the black color). The estimate of the spectral density by the real data was made basing on the period chart of the readings of the scalar magnetometer. We can see form the diagram that the spectral density of the model of the anomalous field was selected sufficiently well.

6. FORMULATION AND SOLUTION TO THE ESTIMATION PROBLEM

Let us write out the vector of state $\overline{x}$:

$$\overline{x} = (K_1, K_2, K_3, \Delta L_{11}, L_{12}, L_{13}, \Delta L_{22}, L_{23}, \Delta M_{11}, M_{12}, M_{13}, M_{21},$$
$$\Delta M_{22}, M_{23}, M_{31}, M_{32}, B^h, B^h_x, B^h_{xx}, B_z, B_{zx})^{\mathrm{T}} = (x_1, \ldots, x_{21})^{\mathrm{T}}.$$

Here the first 16 coordinates are observed parameters of scalar model (5) and the last 5 coordinates are parameters of stochastic magnetic field (7) and (8).

To obtain a closed system of differential equations for the vector x, we shall use the assumption about the consistency of deviation parameters and relation (7) and (8),

$$\frac{dx_i}{dt} = 0 \quad \text{at} \quad i = 1, \ldots, 16,$$
$$\frac{dx_{17}}{dt} = Vx_{18}, \ \frac{dx_{18}}{dt} = Vx_{19}, \ \frac{dx_{19}}{dt} = Vq_1, \ \frac{dx_{20}}{dt} = Vx_{21}, \ \frac{dx_{21}}{dt} = Vq_2, \tag{9}$$
$$M[q_j] = 0, \quad M[q_j(t)q_i^T(s)] = \sigma_j^2\delta(t-s), \quad j = 1, 2.$$

The basis of the model of measurements will be a scalar model of measurements accepting $z = B$:

$$z = x_{17} + x_{20}(h - h_0) + x_1e_1 + x_2e_2 + x_3e_3 + x_4e_1^2 + x_7e_2^2 + x_5e_1e_2$$
$$+ x_6e_1e_3 + x_8e_2e_3 + x_9e_1\frac{de_1}{dt} + x_{13}e_2\frac{de_2}{dt} + x_{10}e_1\frac{de_2}{dt} + x_{12}e_2\frac{de_1}{dt}$$
$$+ x_{11}e_1\frac{de_3}{dt} + x_{15}e_3\frac{de_1}{dt} + x_{14}e_2\frac{de_3}{dt} + x_{16}e_3\frac{de_2}{dt} + r. \tag{10}$$

Posed stochastic estimation problem (9) and (10) is solved by the method of Kalman filtering [8]. In working with real data, V and h are defined according to the information of the GPS receiver;

e_i are determined by the readings of the ferroprobe sensor; the parameters σ_1, σ_2 for q_1, q_2 are selected proceeding from spectral characteristics of the magnetic field 10^{-6}; the parameter σ_r of the noise component of measurements r is selected according to characteristics of the measuring equipment.

To ensure the convergence of the estimate of the anomalous field described by the system of equations, the solution to which is unstable, close points of routes of different directions are processed in addition. It is assumed that the discrepancy of the value of the anomalous magnetic field is directly proportionate to its gradient. Therefore, in calibrating, the presence of the central point through which the routes of the four directions pass, is very important.

7. CONCLUSIONS

The reduction of the problem of magnetometer deviation compensation to the standard formulation of the stochastic estimation problem is studied. An important feature of this approach is the introduction of the stochastic model of the geomagnetic field anomaly. Its parameters were chosen basing on spectral and variance analysis of the aerial survey data.

In the paper, we made normalization of the parameter problem. The normalization made it possible to estimate the accuracy of the compensation, obtain the scalar measurement model of the specified accuracy, and perform the necessary decomposition of the problem.

The posed stochastic problem is solved by the method of Kalman filtering. The aerial survey data are processed basing on the proposed algorithm; some results are published in [9]. The accuracy of the compensation was about 0.1 nT. In one of the variants, the calibration flight was made by the courses 45°, 135°, 225°, and 315° and the survey was conducted by the routes with the courses 0° and 180°.

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